Inventory by the Wavelet Transform

1. The Astronomical Image Inventory

Large astronomical plates are today analyzed with fast scanners leading to images with about 10^9 pixels. This amount of information leads the astronomers to examine sets of computer vision techniques in order to get an available inventory of the contained objects.

The detection is the first kind of the desired results. It is important to be aware of the miss and false alarm probabilities. The principal information lies in the position and in the magnitude of the detected objects. We want to get the most accurate values, with the less bias. We have also to recognize the objects (stars, galaxies, asteroids, etc.) hence we need to get available pattern parameters leading to a nice separation between classes.

In order to get the required information we need a *vision model*. Many kinds of such models have been implemented, the one resulting from the wavelet transform corresponding to a new approach.

2. Some Typical Vision Models in Astronomy

The classical computer vision for robotic and industrial vision is based on the detection of the edges. We have first applied this model to astronomical imagery (Bijaoui et al. 1978). We choose the Laplacian of the intensity as the edge line. As this function is the sum of the second partial derivatives of a noisy function, we need to smooth and to threshold it. The results are independent of large scale variations, such that the ones due to sky background. The resulting procedure is very fast, requiring small memory sizes. No previous background mapping is necessary leading to real time analyses. Many false detections exist if we do not want to miss real objects. The accuracy of the magnitudes is not sufficient. But the main disadvantage lies in the difficulty of getting an available object classification: astronomical sources can not be recognized from their edges, but from their intensity profiles.

Many reduction procedures have been built using a model for which the image is the sum of a slowly variable background with superimposed small scale objects (Stobie, 1986; Slezak et al. 1988). The first step requires the construction of a background (Bijaoui, 1980). For that purpose we need to introduce a scale: the background is defined in a given area. Many statistical estimators derived from the local histogram of intensity are used: mode, median, resulting from a model, etc.

The resulting background map is subtracted. Each pixel which has a significant intensity is considered to belong to a real object. A cross-correlation with the star profile is done, in order to optimize the detection of these objects. A threshold is computed from the distribution of the intensity pixels. An image labelling is performed (Rosenfeld, 1969), bringing positions, magnitudes and pattern parameters.

Generally, this procedure leads to quite accurate measurements, with an available detection and recognition. The computations are fast and require only small computer memory. The model works very well for poor fields, if it is not the case, a labelled field may correspond to many objects. The background map is done at a given scale: larger objects are removed. The smoothing is only adapted to the star detection not to larger objects. The analysis does not take into account the wings of the objects. The classification allows us to separate stars from galaxies but not to recognize the galaxy type.

An improvement of the previous model is done with the introduction of the radial profile of each source (Le Fèvre et al. 1986; Slezak et al. 1988). An astronomical object is associated to a point-like structure. We have thus only to detect the local maxima. Its radial profile contains the main information on the source. The method is similar to the previous one up to the image labelling which is replaced by a maxima detection followed by the determination of the radial profiles. The quality of the measurements is increased, and the derived pattern parameters permits a gain in the separation between the stars and the galaxies.

The defects of this procedure lie in the impossibility of describing complex structures. The method is adapted to quasi stellar sources, on a slowly varying background.

3. A Vision Model with the Discrete Wavelet Transform

In fact the three vision models we used on many sets of images failed to bring about a complete analysis because they are based on a single scale for the adapted smoothing and for the background mapping. The observation of sky images furnishes lots of examples for which we see a small star embedded in a larger structure, itself embedded in a larger one, and so on. A multiscale analysis permits to get a background adapted to a given object and to optimize the detection of different size objects. It is the reason why we were interested in the use of the Wavelet Transform.

Morlet-Grossmann's (Grossmann & Morlet, 1985) definition of the continuous wavelet transform for a 1*D* signal $f(x) \in L^2(R)$ is:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) g^*\left(\frac{x-b}{a}\right) dx$$

z^* designs the conjugate of z. $g^*(x)$ is the analyzing wavelet. a (> 0) is the scale parameter. b is the position parameter.

It is a linear transformation which is essential for numerical algorithms, statistical computation and understanding of the results. The wavelet transform is covariant under translations: the analysis does not depend on the origin of the coordinate frame. It is the general property of convolution operators. It is also covariant under dilatations: this is the property which gives its originality to the wavelet transform. We get a mathematical microscope the properties of which do not change with the magnification.

Our vision model is based on the splitting of the image into scale space allowing us to detect objects of different sizes. The discrete wavelet transform can be processed by many algorithms (Bijaoui 1991). The constraints we put on the transform result from the chosen strategy. Stars, and generally astronomical sources are quite isotropic sources, no direction is privileged. Thus we choose an isotropic wavelet. We need to connect fields from different scales. The redundancy is not critical, but we need to restore an image from the transform. At last, we need also to have a fast algorithm. These constraints led us to use the *Algorithme à trous* (Bijaoui, 1991; Holdschneider et al. 1989) resulting from the difference between two B-spline interpolations. $B_3(x)$ is close to a Gaussian function and the results are quasi-isotropic. With $B_5(x)$ the discrepancy to the Gaussian is very faint, and the interpolation and the wavelet can be considered as isotropic.

4. The Object Definition and the Partial Reconstruction

After applying the wavelet transform on the image, we have to extract, to measure and to recognize the significant structures. The wavelet space is a 3*D* one. An object has to be defined in this space.

In the first step, we do an image segmentation scale/scale in the wavelet space. An object could be defined from each labelled fields, without taking into account the interscale neighbourhood. We can restore an image of these objects from the known wavelet coefficients, but this restoration does not use all the information.

Secondly, we link the labelled fields from a scale to the following one. That leads to building a tree of neighbourhoods, from the largest scale to the smallest one. After this operation we can say if a large scale field contains smaller ones which contains smaller ones, and so on.

The image is a set of connected trees, corresponding to different objects. We could define an object as one tree, but it appears that we reduce in a too high manner the number of objects. A small star may belong to a small nebula, the tree corresponds to the nebula, and we do not consider the star if we take into account only the connected tree. It is the reason why we define an object as a subtree resulting from the image segmentation in the wavelet space.

Let us consider now an object, such as it was defined. It corresponds to a field D in the wavelet space. It is fully determined from its wavelet coefficients $w_0^{(i)}(k,l)$. We have to restore an intensity distribution $c^{(0)}(k,l)$ such that its wavelet transform has the same coefficients in D. The restoration algorithm is an extension of the classical Van Cittert's deconvolution algorithm (Burger & Van Cittert 1932).

This algorithm provides an image for each object. It is easy to compute from each of them any kind of parameters: mean position, total intensity, pattern parameters, etc. Now, our experiments show us that the quality of the detection is very nice in this procedure. A recent experiment on SA57 field gives us a dispersion of less than 0.08 for the magnitudes of about 23–24 compared to another careful interactive processing. Using a very different approach, Coupinot et al. (1992) showed that they get accurate measurements from the wavelet transform.

5. Conclusion

The vision model resulting from the wavelet transform permits us to detect, to measure and to recognize an object as complex as available. We have not done enough experiments to claim that the resulting measurements would be more accurate than the ones derived from other models. The procedure does not introduce any prior information on the stellar profile or on the scale of the background variations. This is very important for automated procedures.

The main disadvantage lies in the number of used data. The algorithm *à trous* leads to an increase in the data by the number of scales. In our experiments we used 4 – 5 scales. The increase is too high for large astronomical images. We are examining now a way to reduce this data amount with a pyramidal transform.

This vision model may be improved using the stellar profile. In the wavelet space, we can recognize the wavelet images connected to star-like objects. The procedure is more complicated and we used it only for the image restoration (Starck & Bijaoui 1992).

References

Bijaoui, A., 1980. *Astron. Astrophys.* **84**. 81-84.

Bijaoui, A., 1991. *Ondelettes et Paquet d'Ondes*, ed INRIA, p. 115-140.

Bijaoui, A., Lago, G., Marchal, J. and Ounnas, C., 1978. *Traitement des Images et Reconnaissance des Formes*, ed. INRIA, p. 848.

Burger, H.C. and Van Cittert, P.H., 1932. *Z. Physik*, **79**, 722.

Coupinot, G., Hecquet, J., Aurière, M. and Futaully, R., 1992. *Astron. Astrophys.*, **259**, 701-710.

Grossmann, A. and Morlet, J., 1985. *Mathematics and Physics, Lecture on Recent Results*. World Scientific Publ.

Holdschneider, M., Kronland-Martinet, R., Morlet, J. and Tchamitchian, P., 1989. In *Wavelets*, ed. J.M. Combes et al. [Springer: Berlin], p. 286-297.

Le Fèvre, O., Bijaoui, A., Mathez, G., Picat, J.P. and Lelièvre, G., 1986. *Astron. Astrophys.*, **154**, 92.

Rosenfeld, A., 1969. In *Picture Processing*. Academic Press, New York. 127.

Slezak, E., Bijaoui, A. and Mars, G., 1988. *Astron. Astrophys.*, **200**, 1-20.

Slezak, E., Mars, G., Bijaoui, A., Balkowski, C. and Fontanelli, P., 1988. *Astron. Astrophys. Sup. Ser.*, **74**, 83-106.

Starck, J.L. and Bijaoui, A., June 1992. *Wavelets and Applications*. Eds Y. Meyer and S. Roques, Toulouse.

Stobie, R.S., 1986. *Pattern Recognition Letters*. 317-324.

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